MANUAL
OF THE
CELESTIAL COORDINATOR

By

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U. S. NAVY

WEEMS SYSTEM OF NAVIGATION
ANNAPOLIS, MARYLAND

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INTRODUCTION

In explaining the various uses of the Celestial Coordinator, an orthographic projection is employed in this treatise. However, the same problems can be solved with a Coordinator on the stereographic projection, in a manner identical to that explained for the orthographic projection.

The purpose of either projection is to represent on a plane surface the appearance of a spherical one. The orthographic projection of each point is obtained by dropping a perpendicular to the surface projected on, as illustrated in Figure "A". This represents how it would appear to an eye located at an infinite distance.

Figure "B" illustrates the stereographic projection of a hemisphere. The projection of each point on one hemisphere is toward the midpoint of the opposite hemisphere.

The orthographic projection of a hemisphere, with a parallel and secondary every fifteen degrees, is illustrated in Figure "C", while a stereographic projection of the same thing is illustrated in Figure "D".

For convenience, the Coordinator is based on the viewpoint being from the westward, so that North (N) is always to the left, and South (S) to the right. This simplifies determining the True Azimuth.

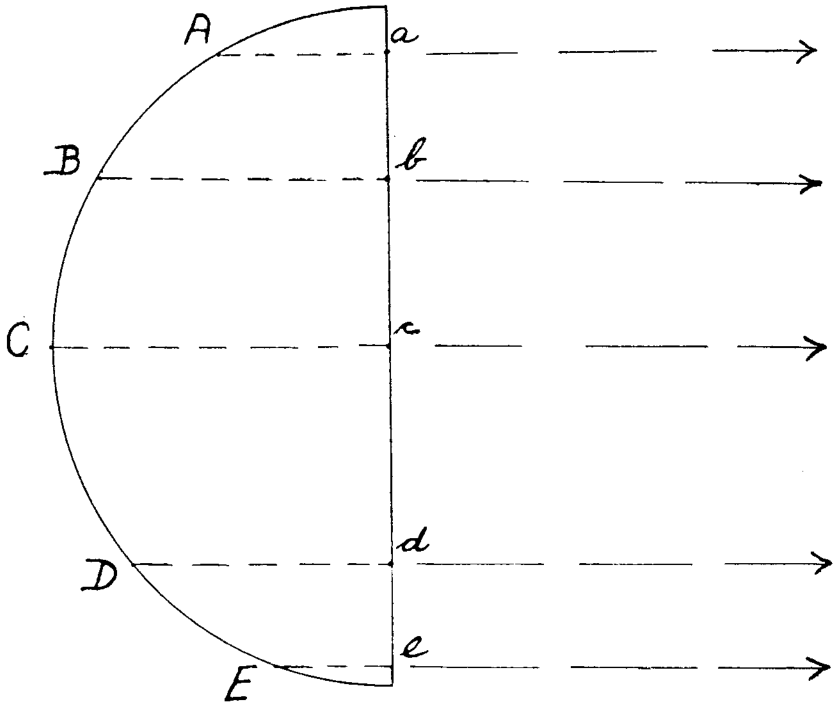

Fig. A

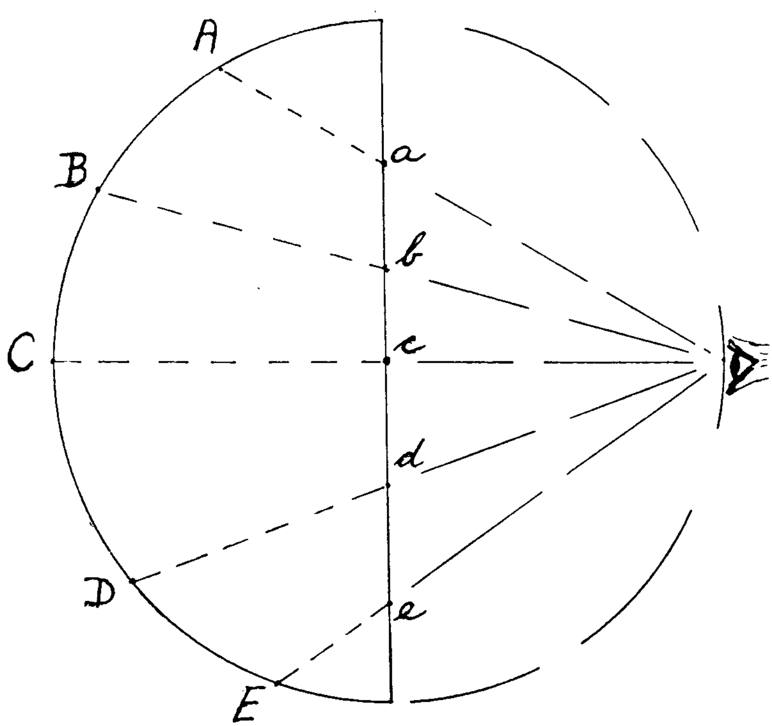

Fig. B

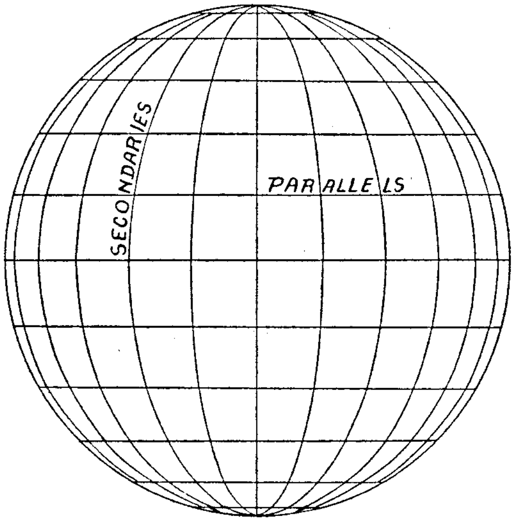

Fig. C

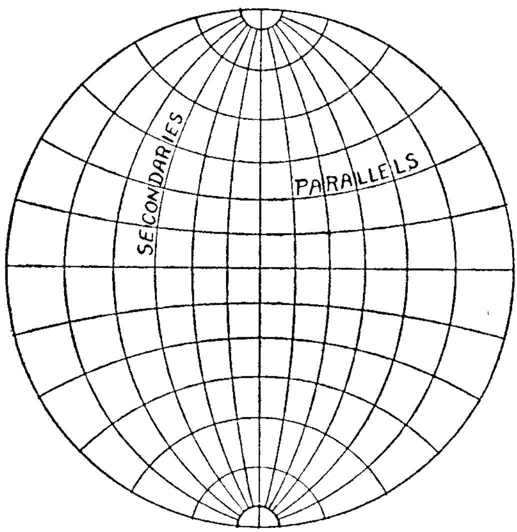

Fig. D

DESCRIPTION

The Celestial Coordinator consists essentially of two equal concentric circles, each marked to represent the projection of a system of spherical coordinates on the plane of a meridian. In this description, the orthographic projection is employed.

The device is constructed primarily for the purpose of providing graphic illustration and approximate solution of the problems involving spherical trigonometry which arise in Celestial Navigation. For the solution of these problems, the circles of the Coordinator represent the Horizon System and the Equinoctial System of coordinates.

In the Horizon System, the fundamental circle is the horizon of the observer. In the Equinoctial System the fundamental circle is the celestial equator, or Equinoctial. The two coordinates of either system provide the locus of a point on the Celestial Sphere, just as Latitude and Longitude are used to designate the location of a given point on the surface of the Earth.

The coordinates of the Horizon System are called Altitude and Azimuth, while those of the Equinoctial System are called Declination and Meridian Angle. These are analogous, respectively in each case, to Latitude and Longitude.

On the Coordinator, the *fixed* portion represents the Horizon System, and the movable disc the Equinoctial System. The following nomenclature pertaining to these two systems should be thoroughly mastered.

THE HORIZON SYSTEM

In order to see the Horizon System without interference, move Pn to Z. In this position the two systems coincide, which would be the case if an observer were located at the North Pole of the earth. Now we can examine the Horizon system in detail, and note the following nomenclature. See Figure 1.

HORIZON: The fundamental circle of the horizon system, represented by the line NS. It must be thoroughly *understood* that *this* and *all other* straight lines on the Coordinator represent *circles.*

ZENITH: The point in the heavens directly overhead, or 90° above the horizon. It is one of the "poles" of the horizon system and is located at "Z" on the Coordinator. The other pole of the horizon system is called the Nadir.

VERTICAL CIRCLES: These are the meridians of the horizon system, and are called *Vertical Circles* because they are perpendicular to the horizon. All vertical circles pass through the Zenith and Nadir.

MERIDIAN OF OBSERVER: That Vertical Circle which passes through the North (N) and South (S) points of the Horizon.

PRIME VERTICAL (P.V.): That Vertical Circle whose plane is perpendicular

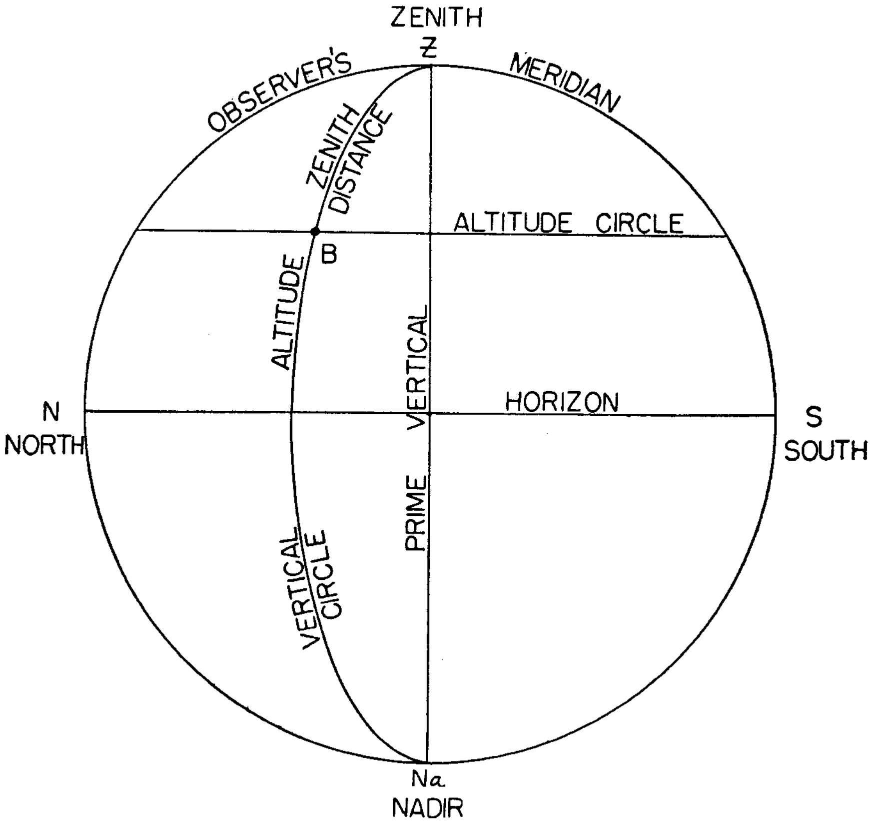

FIG. 1

to the meridian, and which therefore passes through the East and West points of the horizon. It is represented by the vertical diameter on the Coordinator.

ALTITUDE (H): Angular distance above the Horizon; 0° at the Horizon, and 90° at the Zenith. The parallel lines in the horizon system are called *Altitude Circles*. Each one is the locus of all points with the same altitude. The coordinator contains an altitude circle for every 5° up to 70°.

ZENITH DISTANCE (z): Angular distance from the Zenith. It is the complement of altitude. Thus, if altitude is 60°, zenith distance is 30°.

AZIMUTH (Zn): This is the coordinate of the horizon system analogous to Longitude on the earth. It is the angle at the zenith between the meridian of the observer and the vertical circle passing through the body. True Azimuth (Zn) is measured from the North point of the horizon around through East, in arc, from 0° to 360°, which brings it back to North again. Thus, the True Azimuth of East

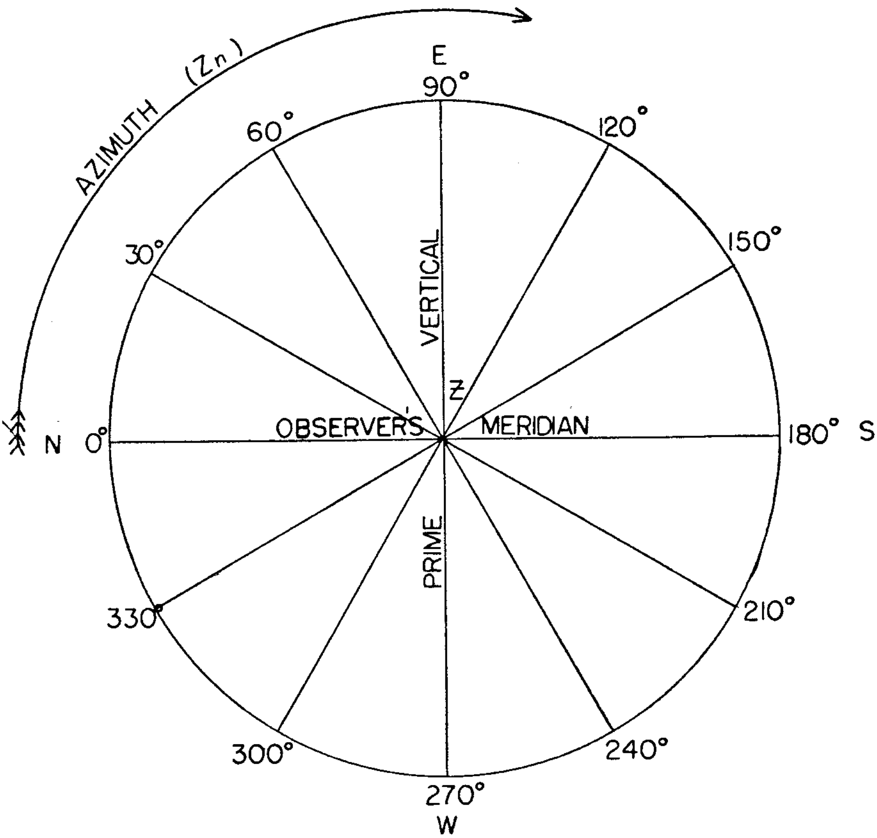

Fig. 2

is 090°, Southeast 135°, South 180°, Southwest 225°, West 270°, Northwest 315°, and so on.

Azimuth is really measured around the horizon, and may be better understood if we transfer our viewpoint over the zenith. The Horizon would then appear as a circle, with Z at the center, and the vertical circles would appear as radii, as illustrated in Figure 2. It will be observed, and this fact must be carefully noted *and remembered,* that when a body is to the *Eastward* of the meridian its Azimuth (Zn) is *less* than 180°. When it is to the *Westward* of the meridian, its Azimuth is *greater* than 180°.

Azimuth is just another name for the *bearing* of a body and is used instead of *bearing* when we are speaking of bodies in the heavens, whereas *bearing* is used to denote the direction of terrestrial objects. It will be noted that the Meridian is the locus of all points whose Azimuths are either 0° or 180°. Likewise, the Prime Vertical is the locus of all points whose Azimuths are either 90° or 270°.

On the Coordinator there is a Vertical Circle for every 5°. Every 10° the vertical circles are heavier than the others. Along the horizon in black letters are shown the *Azimuths* of these heavier circles. All points on a Vertical Circle have the same Azimuth.

The plane surface of the Coordinator represents the entire *surface* of the projected sphere. Thus, while any point on the sphere can project to but one point on the projection, any given point on the projection may

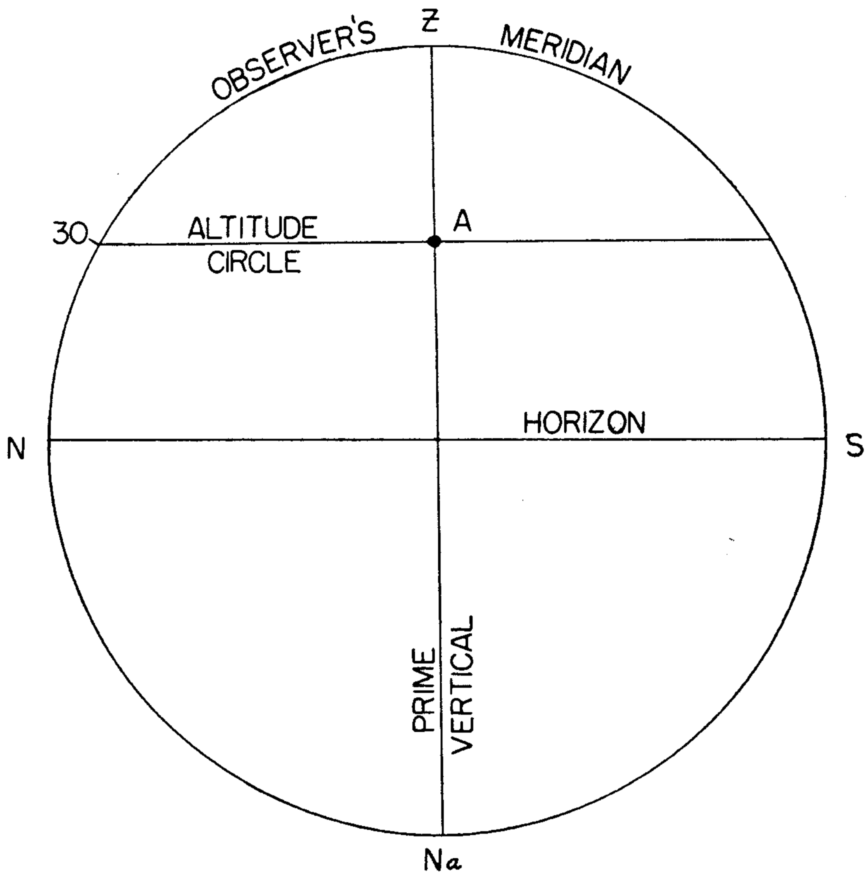

FIG. 3

have come from either of *two* points on the sphere. For example, in Figure 3, the point "A" might be the point whose altitude and azimuth are 30° and 90° respectively, or that point whose altitude and azimuth are 30° and 270° respectively. If we transfer our viewpoint around to S, these two points would appear at A and A′ as illustrated in Figure 4.

The Coordinator is constructed to represent the celestial sphere as seen from the WEST point of the Horizon.

Let us now see how to locate a point whose Altitude and Azimuth are

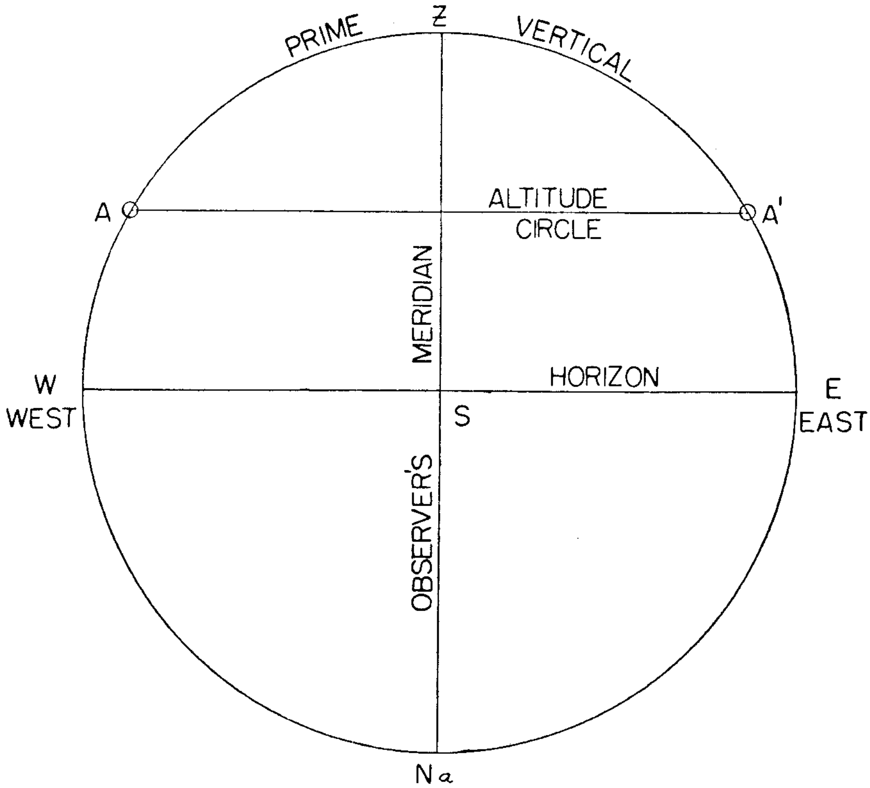

FIG. 4

given: These are the coordinates that a navigator can observe: Altitude with a sextant, and Azimuth with a compass.

Locate the following points on the Coordinator, and decide whether the point is to the Eastward (E) or Westward (W) of the meridian.

	Altitude	*Azimuth*
A	30°	300°
B	45°	120°
C	15°	225°
D	10°	045°

The correct locations and answers are illustrated in Figure 5.

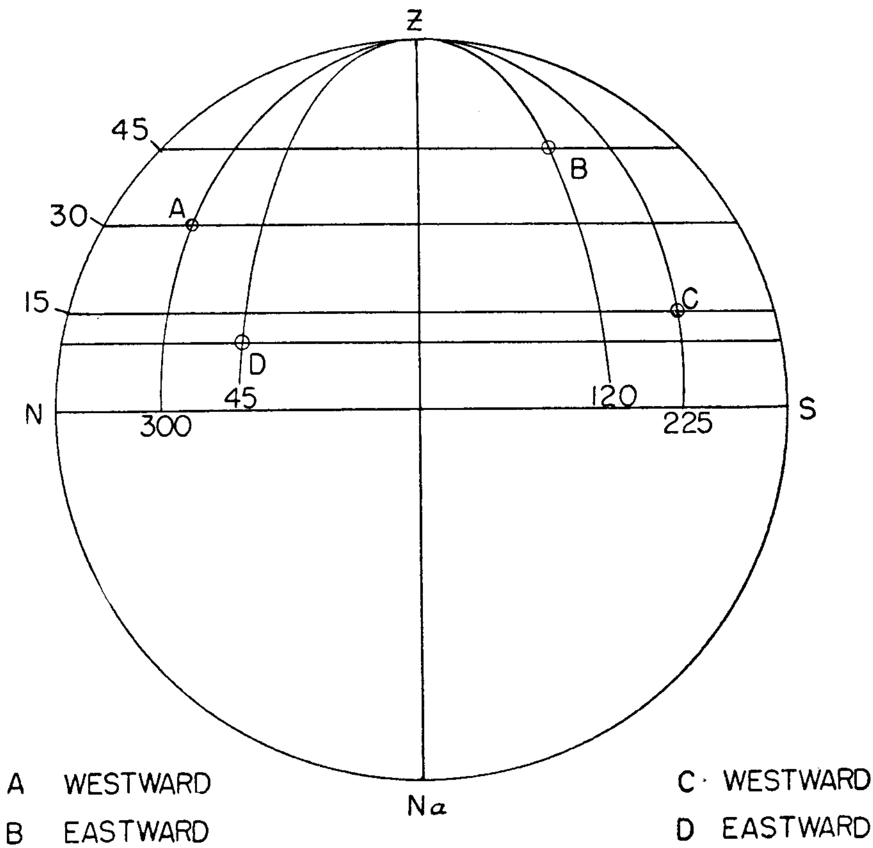
Z
45
30
15
A
B
C
D
N
S
300
45
120
225
Na
A WESTWARD
B EASTWARD
C WESTWARD
D EASTWARD

Fig. 5

On the Coordinator it will be noted that there are *two* numbers representing *Azimuth* for each vertical circle. The *smaller* one is the correct Azimuth if the body is to the *Eastward* of the meridian, and the *larger* if it is to the *Westward.*

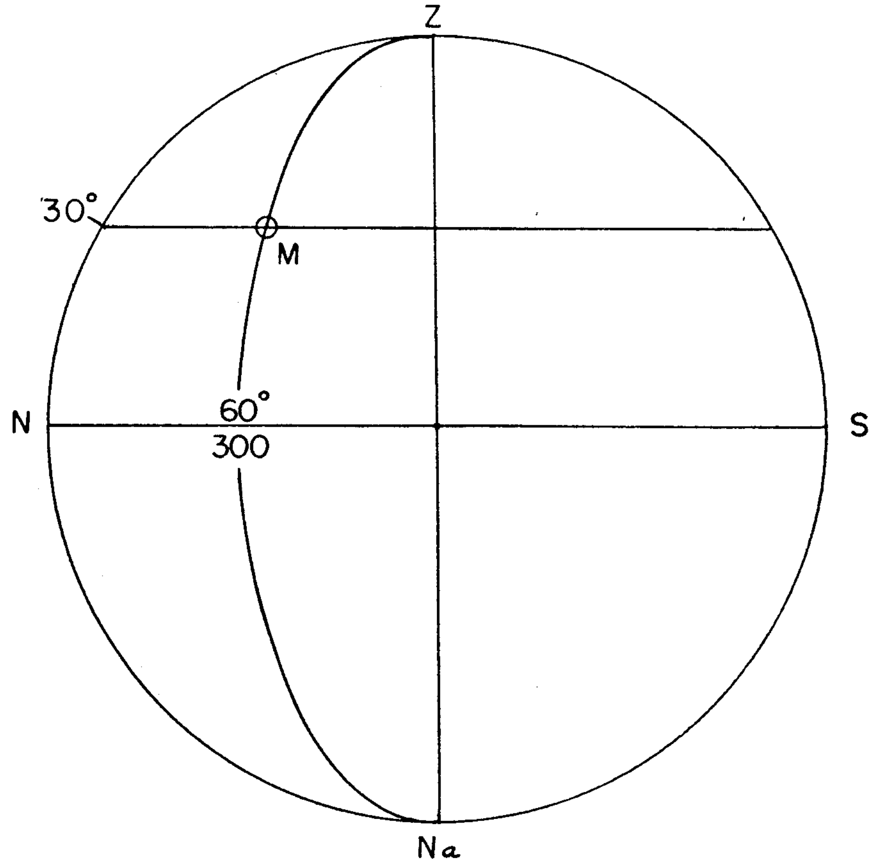

FIG. 6

For example, in Figure 6, there is a body at M. Its altitude is 30°. We cannot tell whether its azimuth is 60° or 300° unless we have some information as to whether the body is to the Eastward or to the Westward of the meridian.

THE EQUINOCTIAL SYSTEM

The coordinates of the Equinoctial System are the terrestrial coordinates of latitude and longitude extended to the Celestial Sphere. However, in the Equinoctial System they are called *declination* and *meridian angle* respectively. Declination, like latitude, is measured North or South from

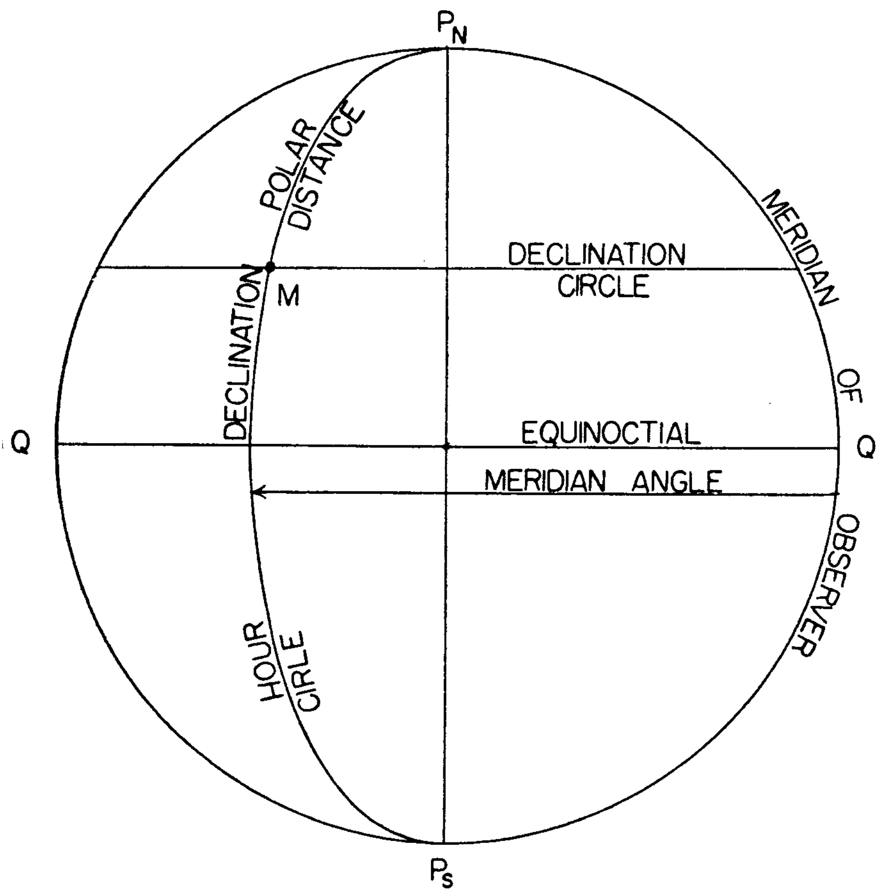

FIG. 7

the equator (Equinoctial). Meridian angle is measured East or West from the *meridian of the observer,* whereas on the earth, longitude is measured in a similar manner from the meridian of Greenwich.

The following nomenclature applies to the Equinoctial System, and is illustrated in Figure 7.

EQUINOCTIAL: The fundamental circle of the equinoctial system, represented by the line QQ′ on the coordinator. As in the horizon system, all straight lines on the coordinator represent *circles.* The Equinoctial is really the earth's equator extended to the celestial sphere.

POLES: The north and south poles of the earth extended to the heavens are represented by the points Pn and Ps respectively, which are the poles of the celestial sphere.

HOUR CIRCLES: The meridians of the equinoctial system are called Hour Circles. All Hour Circles pass through the poles, and are perpendicular to the Equinoctial.

MERIDIAN OF OBSERVER: That Hour Circle which passes through the North and South points of the Horizon, and the zenith and nadir of the observer. It will be noted that the meridian of the observer is both a vertical circle and an hour circle.

DECLINATION (d): This is the coordinate of the Equinoctial System analogous to Latitude on the earth and to Altitude in the Horizon System. It is measured from the Equinoctial toward either pole. It is qualified as *North* when the body lies between the Equinoctial (QQ′) and the North Pole (Pn), or *South* when the body lies between the Equinoctial and the South Pole (Ps). Declination is 0° at the Equinoctial and 90° at the Poles.

POLAR DISTANCE (p): This is angular distance from the elevated Pole. This means that it is measured from the North Pole when the observer is in North Latitude or from the South Pole when the observer is in South Latitude. Its value may be anything from 0° to 180°.

MERIDIAN ANGLE (t): This is the coordinate of the Equinoctial System which corresponds to longitude on the Earth. It is measured from the meridian of the observer to the Eastward (E), or to the Westward (W), depending on whether the body is to the eastward or to the westward of the observer's meridian.

There is shown on the Coordinator an hour circle for every 5° (or 20^m) of meridian angle. Those at intervals of 15° (1^h) are heavier, and are numbered in both arc and time units, starting at the meridian. When a body is *on* the meridian its meridian angle (t) is ZERO. When the meridian angle is 12^h (180°), the body is said to be on the *lower branch* of the meridian.

Any given declination and meridian angle will determine but one point on the Coordinator. However, a given point on the Equinoctial System of the Coordinator may represent either of two points on the sphere.

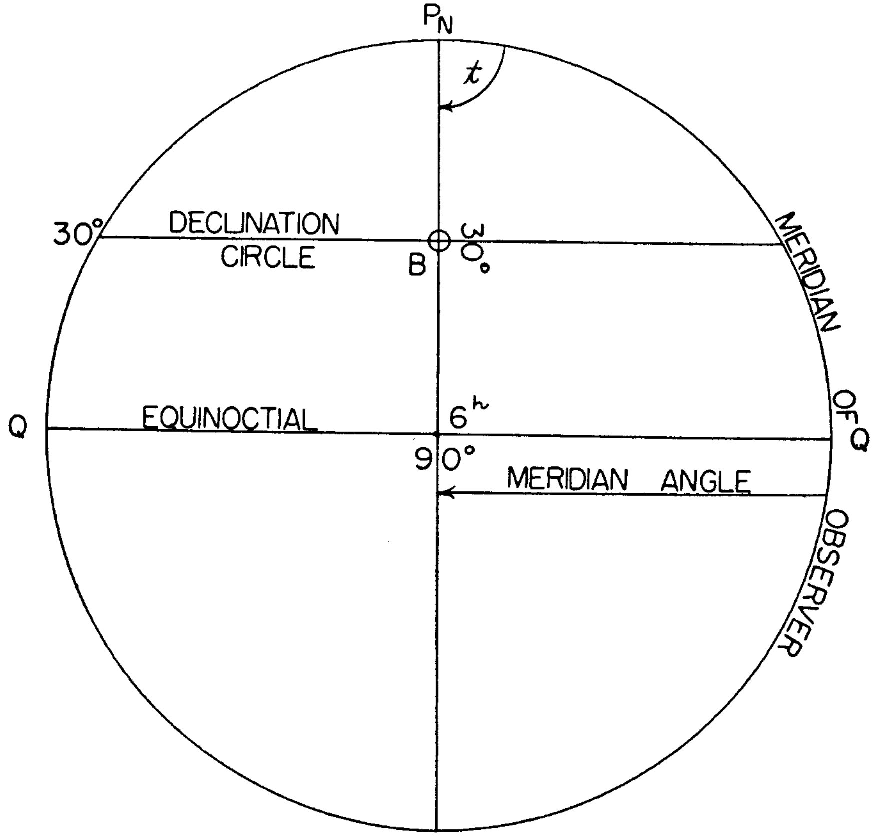

FIG. 8

Figure 8 represents the location of a point whose declination is 30° N., and whose meridian angle is 90° W. However, had the meridian angle been 90° E., the projection would have appeared in the same point. Moving our viewpoint around into the plane of the meridian, these two points would appear at B and B′, as illustrated in Figure 9.

Now we come to the coordination of these two systems of spherical coordinates. The coordination is accomplished by taking into consideration the Latitude of the observer. For an observer at the equator, where the latitude is 0°, the North Pole (Pn) would be at the North point of the Horizon (N), and the South Pole (Ps) at the South point of the Horizon (S). Orient the Equinoctial disc to this position. Now it follows that should the observer move toward either pole, *that* pole would rise above the horizon by an amount equal to the observer's change in latitude.

This brings us to the following proposition: The Latitude of the observer is equal to the Altitude of the Elevated Pole. To orient the Co-

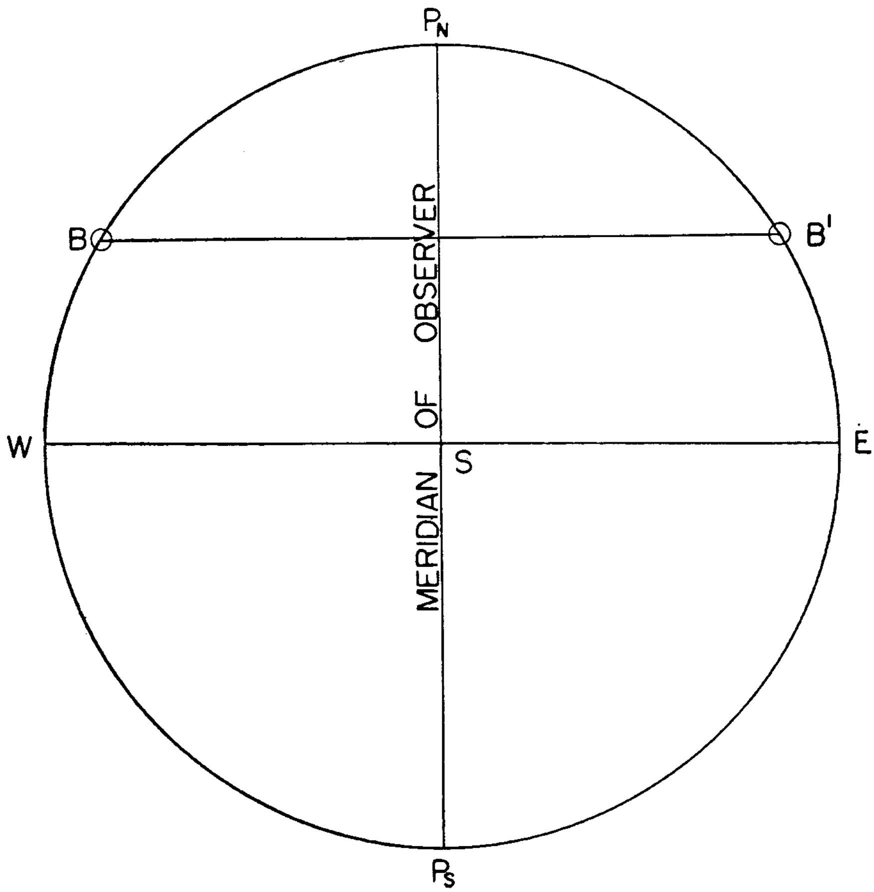

FIG. 9

ordinator for any North latitude set the North Pole (Pn) at an altitude equal to the given Latitude. For South Latitudes, do the same with the South Pole (Ps).

Now we are ready to perform our first Celestial Conversion, from one system of coordinates to the other.

CONVERSION OF COORDINATES

I GIVEN: Latitude (L) 45° N.
Altitude (H) 30°
Azimuth (Zn) 125°

REQUIRED: Declination (d)
Meridian angle (t)

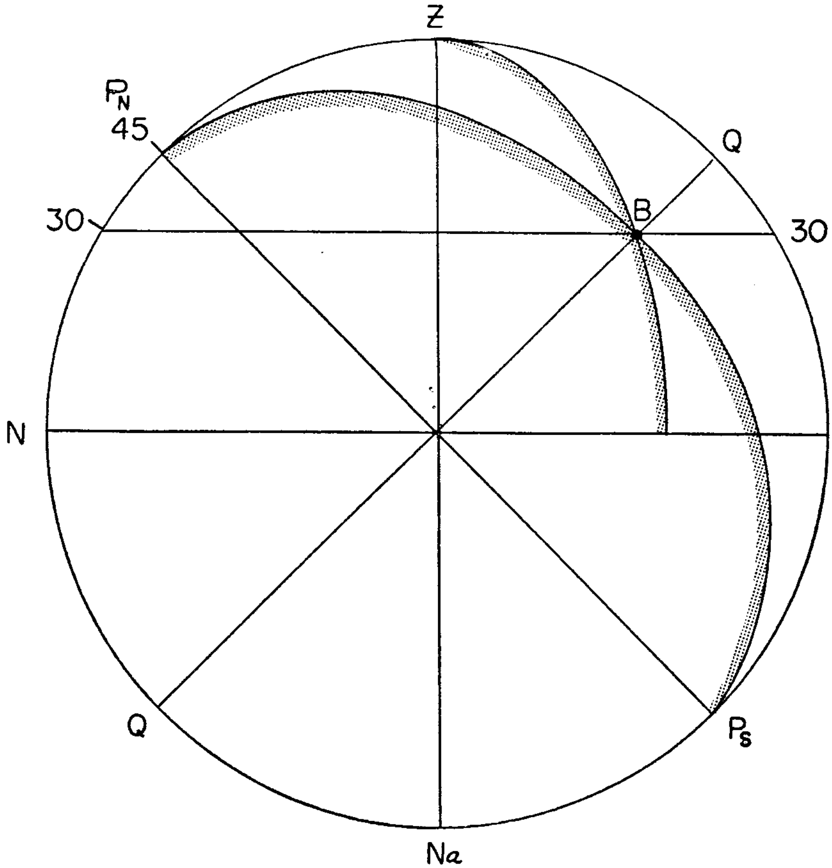

FIG. 10

SOLUTION: Set Pn at 45°. Starting at the horizon, move up the vertical circle of 125° azimuth until you reach the 30° altitude circle. Pick off the Equinoctial system coordinates of this point. Since the Equinoctial itself passes through this point, the declination is 0°. The hour circle of 45° or (3^h) also passes through it. The meridian angle (t) is therefore 45° or 3^h. It remains to decide whether this meridian angle should be called *East* or *West*. The given Azimuth determines this question. Since the Azimuth is 125° (or *less* than 180°) the body is to the *Eastward* of the meridian, and the meridian angle is therefore 45° (or 3^h) *East*. This problem is illustrated in Figure 10.

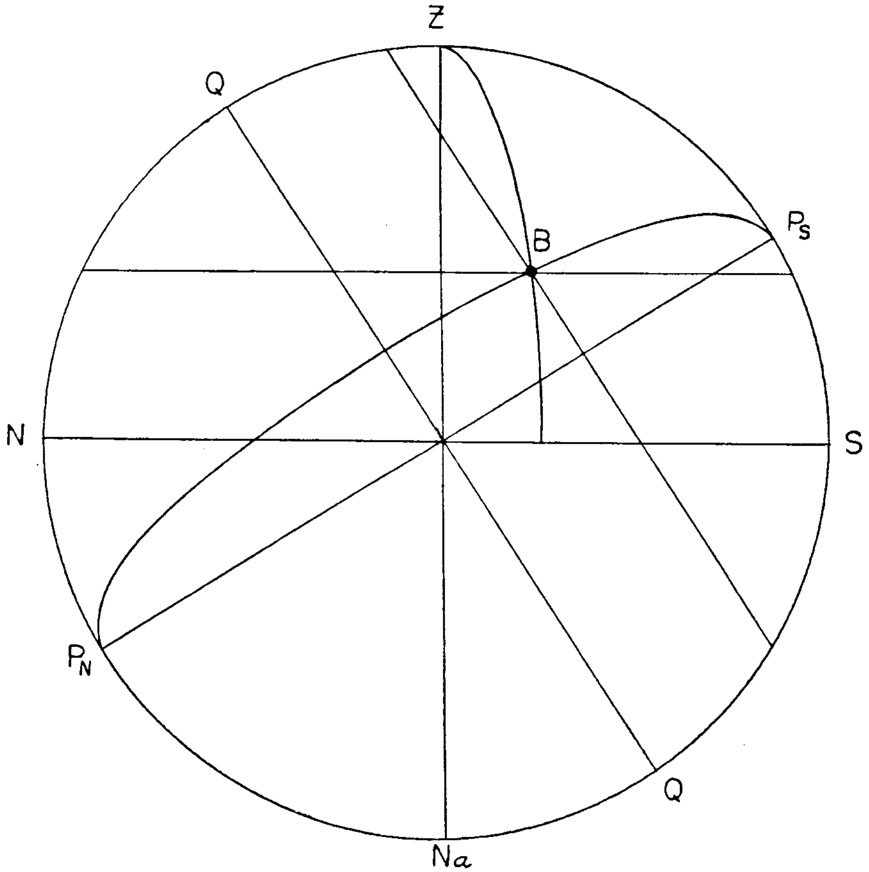

FIG. 11

II GIVEN: Latitude (L) 32° S.
Declination (d) 25° S.
Meridian angle (t) 75° W.

REQUIRED: Altitude (H)
Azimuth (Zn)

SOLUTION: Set Ps at an altitude of 32°. Locate the declination circle of 25° S., and follow it until it crosses the Hour Circle of 75°. From the Horizon System, pick off the altitude (H) and Azimuth (Zn) of the point thus determined. The altitude (H) is found to be 25°. The Azimuth (Zn) might be either 105° or 255°. However, since the meridian angle (t) was given as 75° *West*, the body is to the *westward* of the meridian, and the correct Azimuth (Zn) is 255°. This problem is illustrated in Figure 11.

For practice in the use of the Coordinator, solve the following problems.

I Solve for H and Zn, given:

	L	d	t	H (ANSWERS)	Zn (ANSWERS)
(a)	35° N	10° S	55° E	21.5°	120°
(b)	22° N	35° N	75° W	24°	300°
(c)	55° N	40° N	115° E	20°	047.5°
(d)	27° N	17° S	45° W	28°	230°
(e)	11° S	10° N	40° W	45°	297°
(f)	44° S	50° S	100° E	27°	135°
(g)	63° S	40° S	65° E	46°	090°
(h)	27° S	25° N	40° W	25°	320°

II Solve for "d" and "t", given:

	L	H	Zn	d (ANSWERS)	t (ANSWERS)
(a)	45° S	20°	050°	11° N	47° E
(b)	18° S	40°	250°	27° S	53° W
(c)	25° S	30°	335°	30° N	25° W
(d)	51° S	25°	140°	50° S	115° E
(e)	4° N	55°	240°	13° S	30° W
(f)	57° N	30°	320°	51.5° N	117.5° W
(g)	33° N	55°	090°	27° N	40° E
(h)	8° N	50°	250°	6.5° S	37° W

The problem of solving for H and Zn from "t", "d", and L, is the navigational problem commonly encountered in solving a sight for a line of position.

The problem of solving for "t" and "d", from L, H, and Zn arises in navigation when it is desired to determine the name of a body whose altitude and azimuth have been observed, but whose identity is not known.

There is another problem in navigation, not greatly used in modern times, but which was quite extensively employed for many years. It is called the "time sight", with L, H, and "d" given, to solve for "t" (and if desired, Zn), though the usual solution by logarithms stops with the determination of "t". An example of this type will be illustrated.

GIVEN: L 34° N.
H 45°
d 40° N.

REQUIRED: t and Zn.

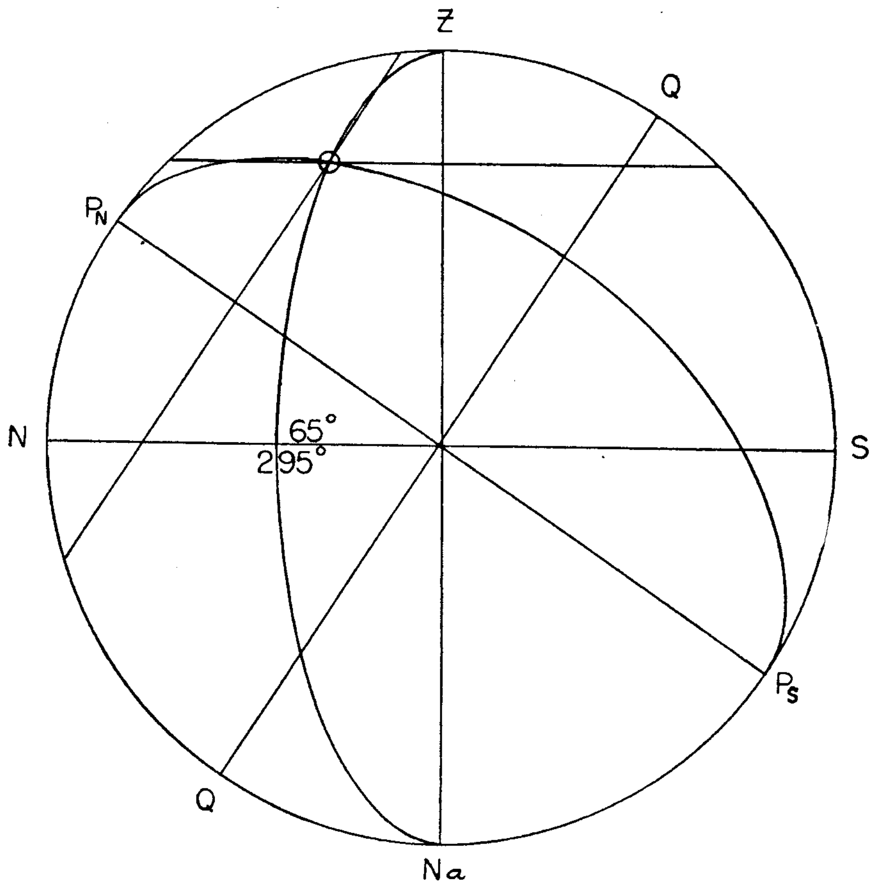

FIG. 12

SOLUTION: Set Pn at 34°. Note the intersection of the 45° altitude circle with the declination circle of 40° N. The resulting "t" is 57°, and the Zn either 65°or 295°. If "t" is 57° *E.*, Zn is 65°, but if "t" is taken as 57° *W.*, then Zn is 295°. To avoid a double solution the observer must know whether the body is to the eastward or to the westward of his meridian. This is usually known in practice. This problem is illustrated in Figure 12.

III Solve for "t" and Zn, given:

	L	d	H		t	Zn
(a)	50° N	20° N	51°	(West)	30° W (2^h00^m)	230°
(b)	42° S	54° S	23°	(East)	110° E (7^h20^m)	143°
(c)	36° N	10° S	31°	(East)	39° E (2^h36^m)	134°
(d)	21° S	25° N	40°	(West)	20° W (1^h20^m)	336°

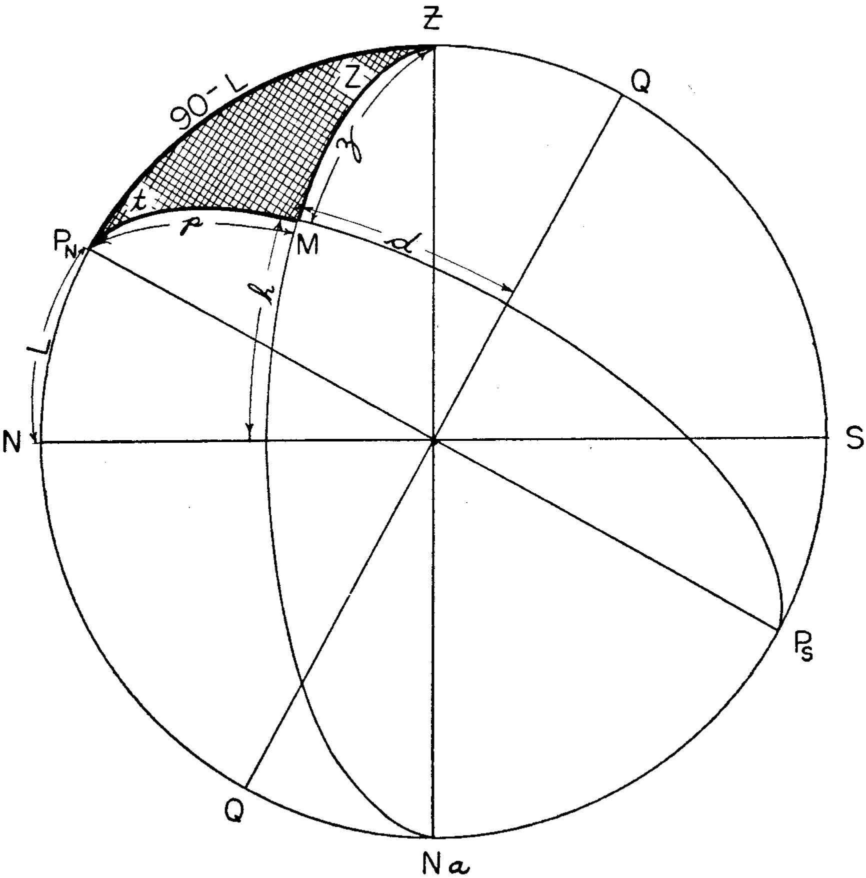

FIG. 13

THE ASTRONOMICAL TRIANGLE

The spherical triangle on whose solution problems in Nautical Astronomy depends, is called the Astronomical Triangle. Its sides are as follows, as illustrated in Figure 13:

ZPn—That part of the observer's meridian from the zenith to the elevated pole. This is equal to 90°—Lat.

ZM—That part of the vertical zircle, from the zenith to the body (M). This is the "Zenith distance" of the body, or 90°—Altitude.

PnM—That part of the hour circle from the elevated pole to the body. This is the "polar distance" of the body, or 90° ± declination.

It is interesting to note that not one of these sides consists of the quantities with which we deal, namely Latitude, altitude and declination. However, by the use of trigonometric co-functions, mathematical solutions may be arranged in terms of L, H, and d, except when the haversine (which has no co-function) is employed.

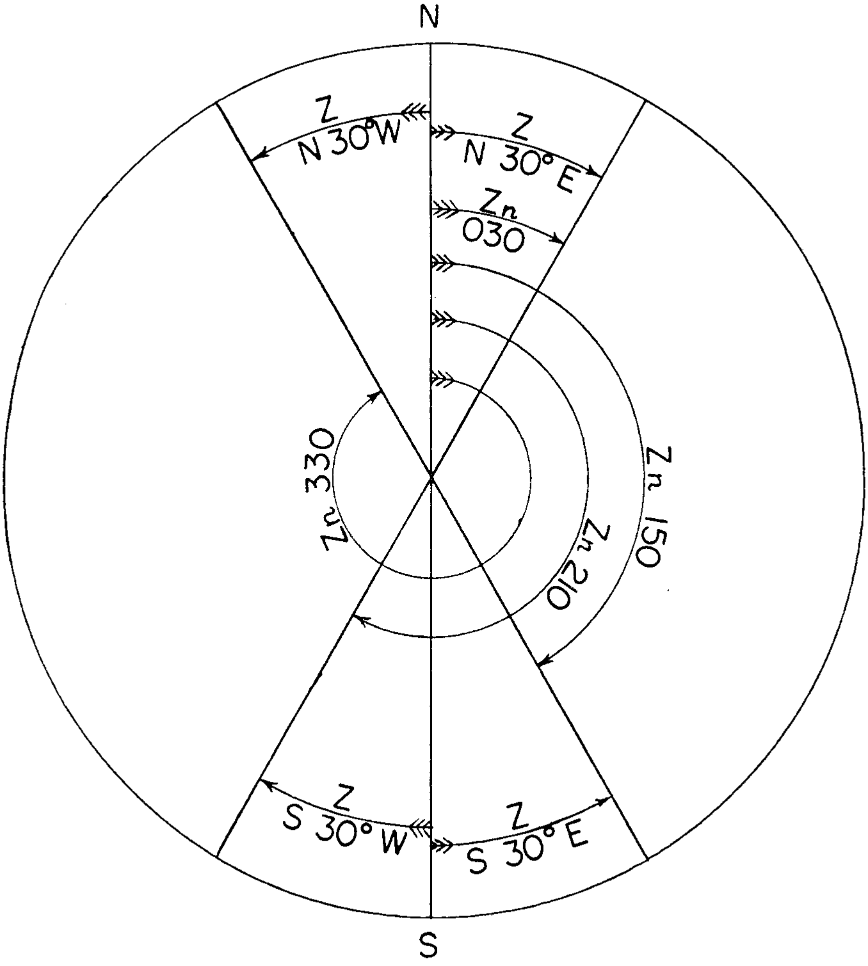

FIG. 14

The angles of the astronomical triangle are "t", "Z", and the angle at M, called the "position angle", which has no significance in navigation. While "t" is always the meridian angle, the angle Z, called "Azimuth Angle" is not the same as true Azimuth (Zn). In all methods of solution except the Coordinator, the *Azimuth Angle* is obtained from the solution, and this must be converted into true Azimuth (Zn) in accordance with definite rules. (The Coordinator gives Zn direct.)

In order to Convert Azimuth Angle (Z) into True Azimuth (Zn), the numerical value of Z must be preceded by a designation of North (N) or South (S), depending on whether the Latitude is North or South, and followed by East (E) or West (W), depending on whether "t" is East or

West. Thus an Azimuth angle of 30° may produce any one of *four* true Azimuths, as follows:
(Figure 14).

If Z = *N* 30° *E*,	Zn = 030° (0° + 30°)
If Z = *N* 30° *W*,	Zn = 330° (360° − 30°)
If Z = *S* 30° *E*,	Zn = 150° (180° − 30°)
If Z = *S* 30° *W*,	Zn = 210° (180° + 30°)

THE POSITION ANGLE

The angle at the body (M), between the vertical and hour circles passing through the body, is called the position angle. While this angle is not used in navigation, it is interesting to note here that its value may be determined readily on the Coordinator by interchanging the values of Latitude and declination, while leaving meridian angle the same. Under this condition the new azimuth angle is equal to the desired angle at M in the original triangle.

The quantity Δ d used in H.O. 214, which represents the difference in altitude due to a 1′ difference in declination (from the tabulated value), is equal to the cosine of this angle M. The student may enjoy verifying this by solving for M on the Coordinator, and looking up its natural cosine in suitable tables.

FOLLOWING THE SUN

The declination circles represent the daily paths of heavenly bodies, due to the rotation of the earth on its axis. For any given latitude, all these paths (sometimes called diurnal circles), make the same angle with the Horizon. (Figure 15).

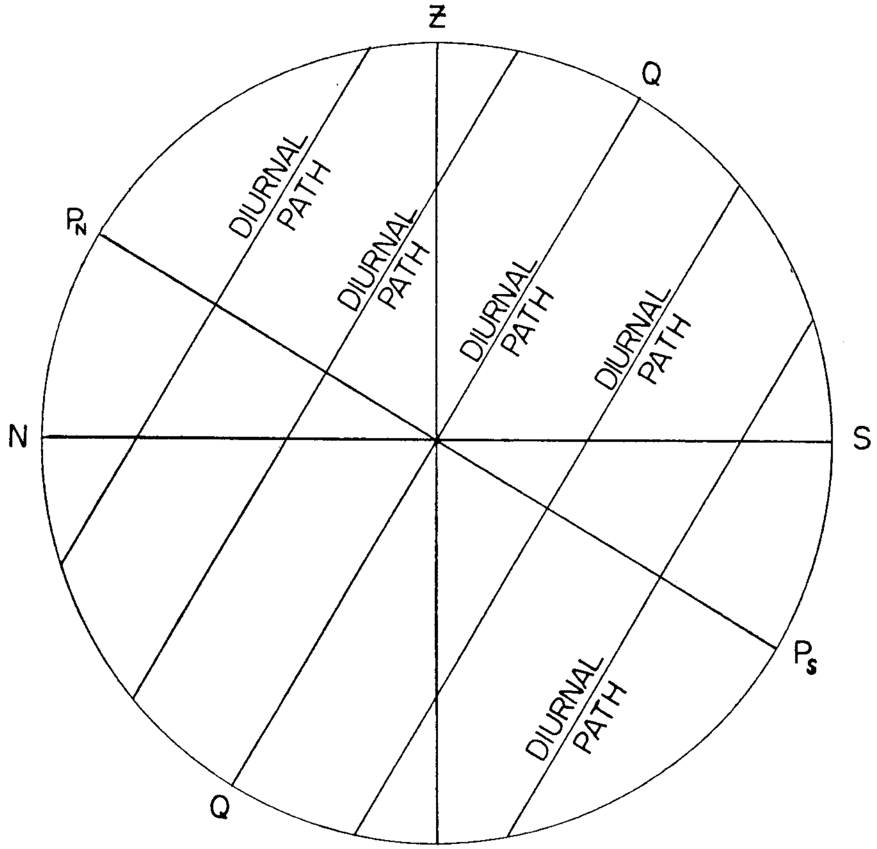

Fig. 15

Since we keep time on the SUN, it is easier to picture the motions and positions of this body. Since the Sun *changes* in declination, it presents a variety of conditions, which we shall now examine.

Let us put ourselves in Latitude 44° N., and orient the Coordinator accordingly. All references to TIME are in terms of LOCAL APPARENT TIME, (LAT) which is all the Coordinator itself can provide, and by SUN we mean the center of the true sun.

On March 21, when the declination of the sun is 0°, its daily path is as indicated in Figure 16. At midnight it is at A; at 6:00 am it is in the celestial horizon at B, and hence just rising. Continuing to rise throughout the forenoon, it reaches C at 8:00 am, and D at 12:00 noon. This point marks its highest altitude. From 12:00 it retraces the same path ar-

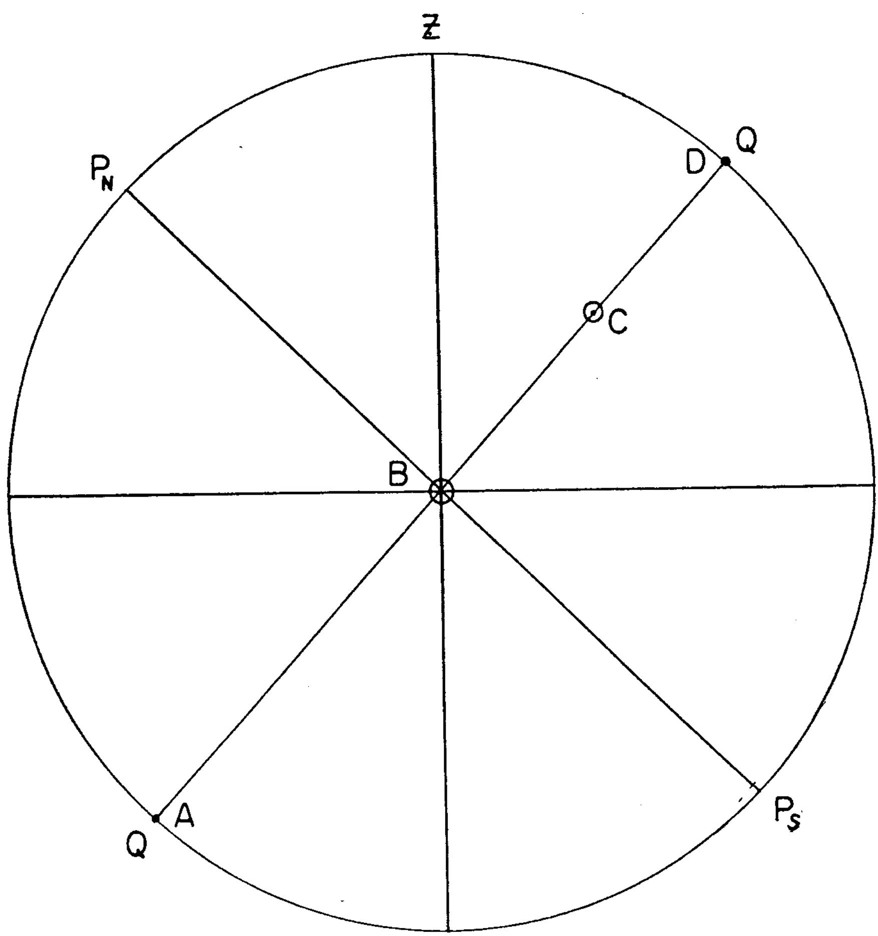

FIG. 16

riving at C at 4:00 pm, setting at B at 6:00 pm, and arriving at A again at midnight.

Note that the time the Sun spends above the horizon (day) is equal to the time spent below it (night). Vary the latitude, and you will observe that when the sun's declination is 0°, the DAY equals the NIGHT, regardless of the Latitude. Now return to Latitude 44° N.

By around May 1, the Sun's declination is about 14° N. Now the daily path is indicated by the line A B C D E in Figure 17. It is at A at midnight, and rises at B at about 5:00; arriving at C at 6:00 am, D at 8:00

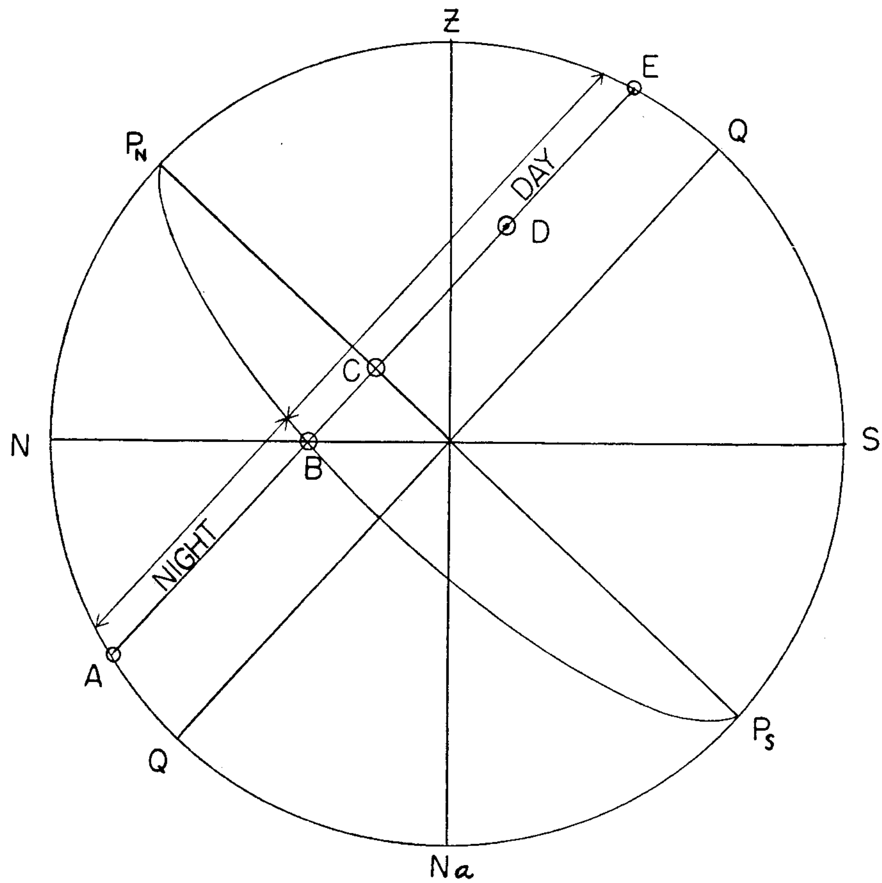

FIG. 17

am, and finally reaches the meridian at E, at noon. From there it passes successively through D at 4:00 pm, C at 6:00 pm, B (SUNSET) at about 7:00 pm, and reaches A again at midnight. The DAY is 14 hours long, and the NIGHT 10.

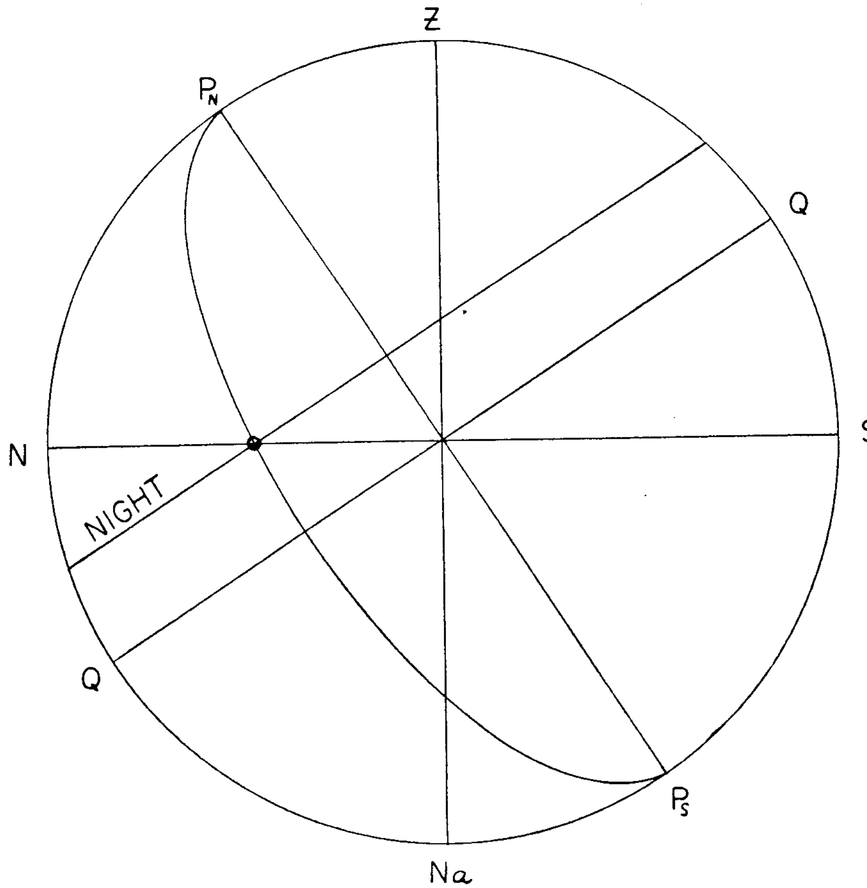

Fig. 18

Now, any change in the latitude will cause a change in the times of Sunrise and SUNSET, and consequently in the relation between the lengths of DAY and NIGHT. At Lat 57° N. on the same date, sunrise would occur at about 4:20 am, and sunset at about 7:40 pm (local apparent time). The DAY would equal about 15^h-20^m and the NIGHT about 8^h-40^m. (Figure 18.)

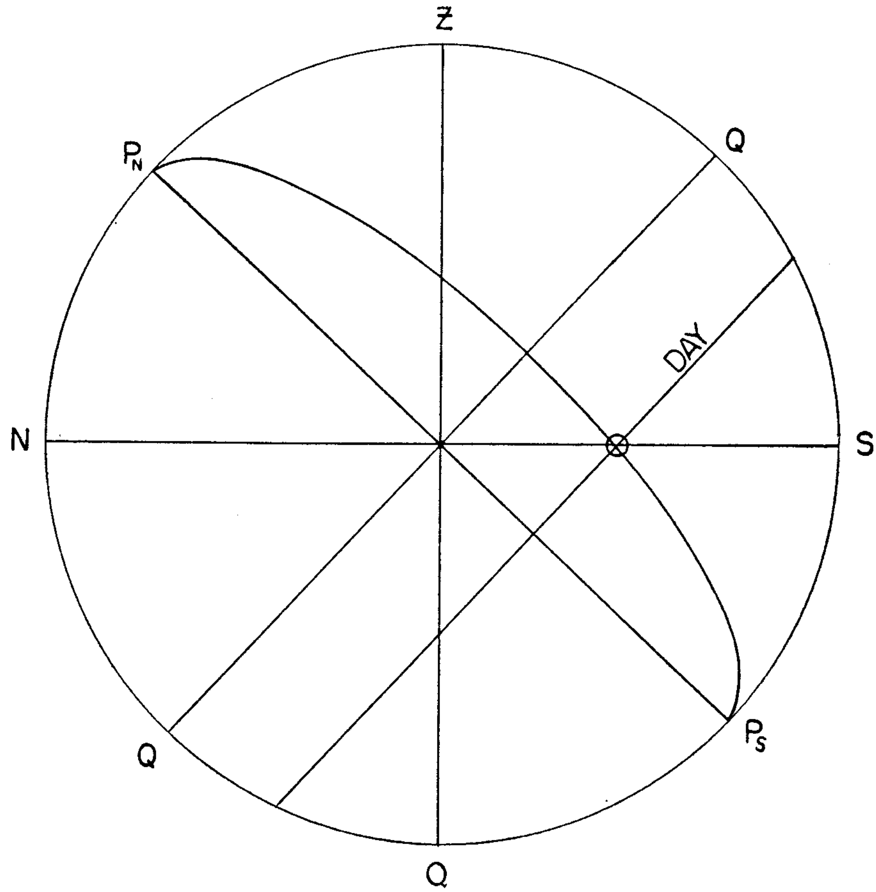

FIG. 19

When the SUN goes into SOUTH declination, northern latitudes experience short DAYS and long NIGHTS. Take for example a place in 43° N. latitude, when the sun's declination is 20° S. It will be seen that sunrise does not occur until about 7:20 am, while sunset comes at about 4:40 pm. This makes the DAY but 9^h-20^m and the NIGHT 14^h-40^m. (Figure 19.)

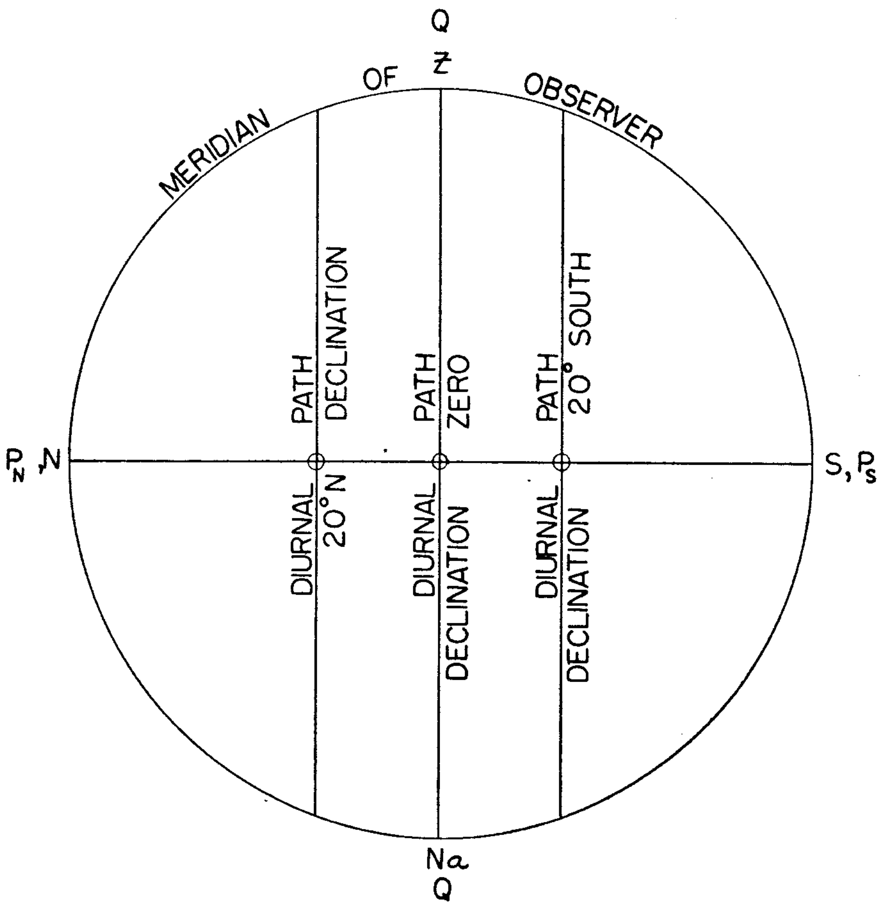

FIG. 20

Let us turn for a moment to the Equator, Latitude 0°. It will be seen that throughout the entire year, regardless of the sun's declination (Figure 20), sunrise is always at 6:00 am; and sunset at 6:00 pm., and that the lengths of DAY and NIGHT are equal throughout the year.

On the Coordinator, the faint lines on the equinoctial system at declinations of 23½ N. and 23½ S. represent the limits of the sun's range in declination.

In order to visualize the travel of the sun for any particular day, its approximate declination must be known, as well as the Latitude of the

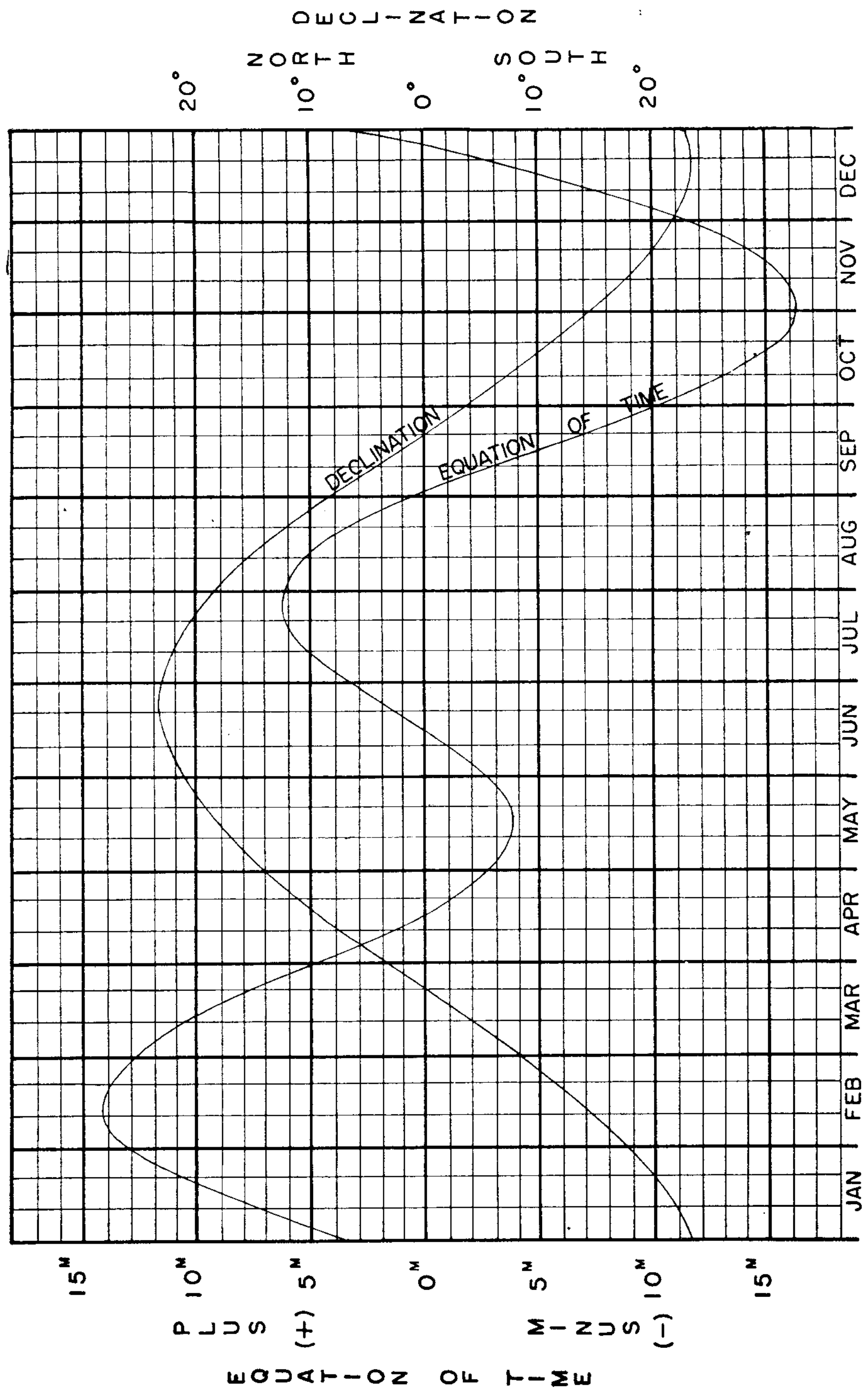
DECLINATION
NORTH
SOUTH
20° 10° 0° 10° 20°
DEC NOV OCT SEP AUG JUL JUN MAY APR MAR FEB JAN
DECLINATION
EQUATION OF TIME
15M 10M 5M 0M 5M 10M 15M
PLUS (+)
MINUS (−)
EQUATION OF TIME

Fig. 21

observer. The diagram illustrated in Figure 21 provides a means of determining the approximate declination of the sun for any day in the year. A line from the desired date, perpendicular to the graduated scale, gives this information. As illustrated in the figure, the sun's declination on May 1 is about 14° North.

This permits us to solve the problem of determining the approximate local Apparent Time of Sunrise and Sunset for any day in the year, in any Latitude.

EXAMPLE: 1. Determine the Local Apparent Time of sunrise in Lat. 40° N on May 21st.

SOLUTION: Set Pn at 40°. From Figure 21 it is seen that the sun's declination is about 19° N. The LAT of sunrise is therefore about 4:50 am.

Additional Problems and Answers

Determine the LAT of Sunrise and sunset in the Latitudes given, for the dates indicated.

		Answers	
Lat.	*Date*	*LAT Sunrise*	*LAT Sunset*
(1) 30° N	Nov. 22	6:50 am	5:10 pm
(2) 22° S	May 1	6:25 am	5:35 pm
(3) 45° N	April 16	5:20 am	6:40 pm
(4) 46° S	Oct. 6	5:40 am	6:20 pm

These results may be checked with the Local Civil Times (LCT) given in the Almanac by making the following corrections to the above:

(1) Minus (−) 5^m to all sunrises and plus (+) 5^m to all sunsets.

This is necessary because the Coordinator gives the time the center of the Sun is in the celestial horizon, while actual sunrise is the instant the *upper limb* reaches the visible horizon. This correction varies with latitude and to some extent with the sun's declination. However, 5^m is about the average.

(2) A correction for the "Equation of time," which may be obtained from the curve in Figure 21. Apply the correction to the LAT in accordance with the sign indicated in the Figure.

The LCT's of sunrise and sunset then become as follows, which should check within a minute or so with the LCT's given in the Almanac for *any* year.

LCT Sunrise	:	LCT Sunset
(1) 6:50 − 5 − 14 = 6:31 am	:	5:10 + 5 − 14 = 5:01 pm
(2) 6:25 − 5 − 3 = 6:17 am	:	5:35 + 5 − 3 = 5:37 pm
(3) 5:20 − 5 + 0 = 5:15 am	:	6:40 + 5 + 0 = 6:45 pm
(4) 5:40 − 5 − 12 = 5:23 am	:	6:20 + 5 − 12 = 6:13 pm

TWILIGHT

There are periods before sunrise and after sunset when visibility is fairly good. These periods are called "Twilight". There are three kinds of twilight, namely "civil", "nautical", and "astronomical".

Civil evening twilight is the interval between sunset and the time the sun reaches a point 6° below the horizon. Morning twilight is the corresponding period before sunrise.

Nautical evening twilight ends when the sun reaches 12° below the horizon, and astronomical evening twilight ends when the sun reaches 18° below the horizon.

The faint lines at 6°, 12°, and 18° below the horizon on the Coordinator facilitate the computation of the lengths of the various kinds of twilight. The Latitude of the observer and the declination of the Sun must be known.

GREAT CIRCLE SAILING

Problems in Great Circle Sailing can be solved completely on the Coordinator, not only for initial Course and distance, but also for Latitude and Longitude of the Vertex, and coordinates of points on the great circle track.

To illustrate the method, the following problem will be solved:

GIVEN: "A" in { Lat. 35° N = L_1; Long. 125° W = λ_1 }

"B" in { Lat. 21° N = L_2; Long. 160° W = λ_2 }

DETERMINE:
(1) Initial Great Circle course, "A" to "B."
(2) Great Circle Distance.
(3) Longitude of Vertex.
(4) Latitude of Vertex.
(5) Latitudes and longitudes of points on the Great Circle Track, using longitudes which are multiples of 5° from the vertex.

SOLUTION: Difference of longitude, (DLo) = 35° W
Set Coordinator, using as Latitude, L_1
Set Coordinator, using as Declination, L_2
Set Coordinator, using as meridian angle, DLo.

Solve for Zn, which is the initial Course, and Zenith distance, which is great Circle distance. Setting the Coordinator for the values indicated, gives the following results: (Figure 22).

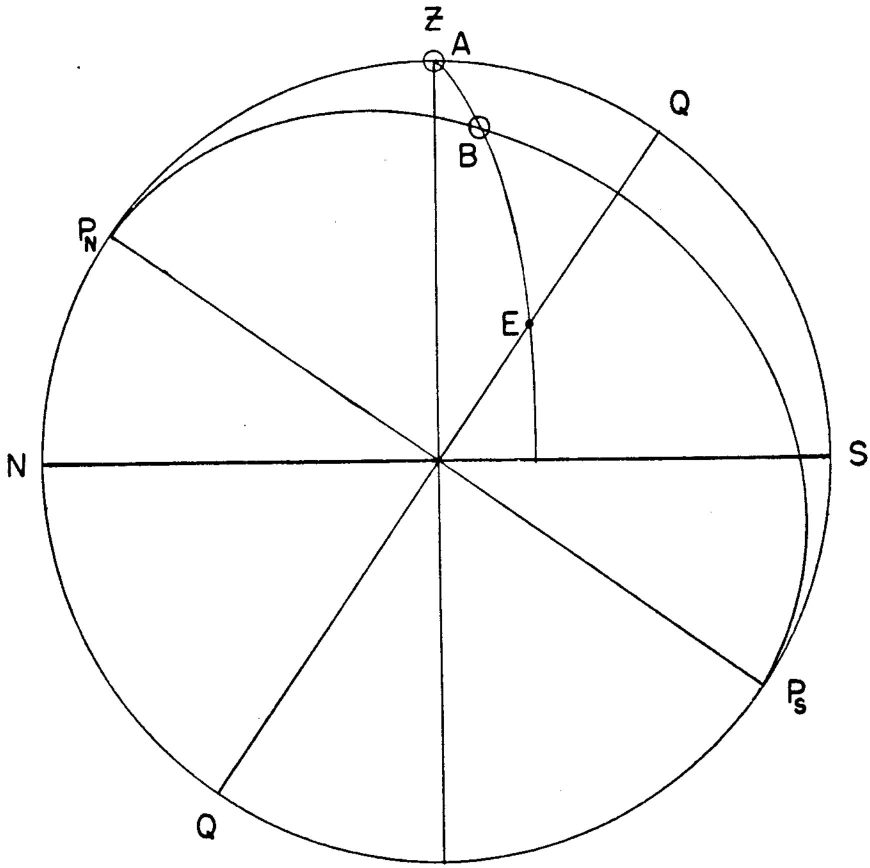

FIG. 22

(1) Initial Course (Zn) = 255° (since DLo is W).
(2) Great Circle Distance (z) = 34° = 2040 miles.

The vertical Circle whose azimuth is 255°, represents the great circle track, with Z the point of departure. Following this vertical circle to the equator, it is seen that the great circle crosses the equator about 65° in longitude (indicated by the value of the meridian angle at this point) to the Westward of A, at a point we shall call E.

We now re-orient the Coordinator to illustrate how this problem looks on the earth's surface. Set Pn at 0°. (Pn now represents point E.)

The Horizon system now represents the Earth. Z is the North pole, the vertical circles are meridians of longitude, and the altitude circles are parallels of latitude.

On the equinoctial system, the hour circles now represent the various great circles which will cross the equator at the same points where our

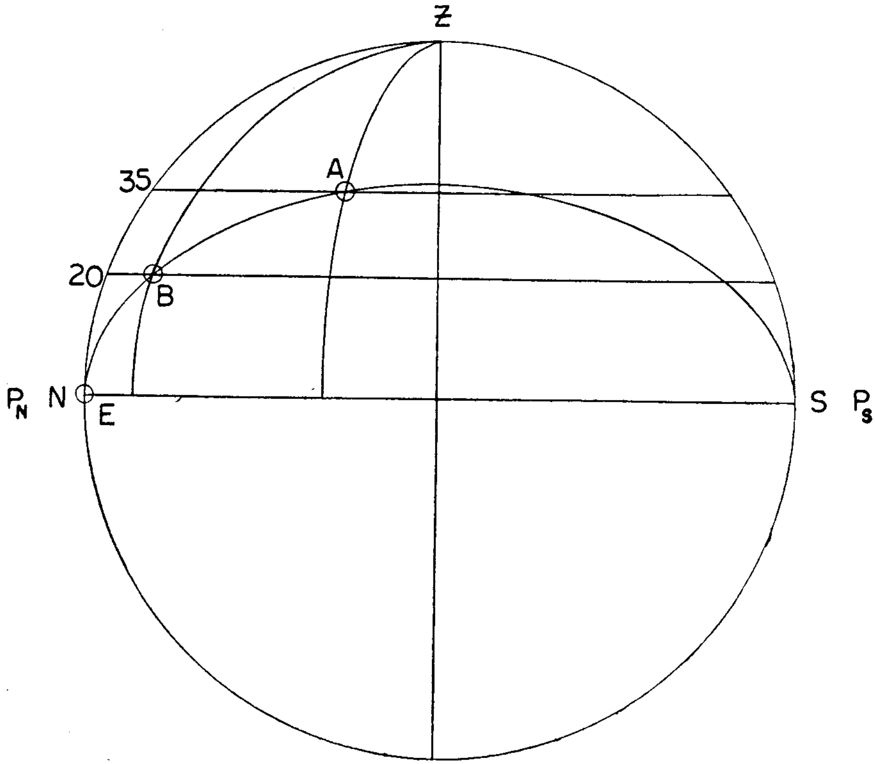

FIG. 23

great circle crosses it (at E). Plot the locations of A and B, by their latitudes as altitude, and their longitudes *from E* as Azimuth.

Thus A is located as Altitude 35° and Azimuth 65°, while B is at Altitude 21° and Azimuth 30°. These points may be plotted, and the Great Circle between them drawn in by hand. It will be seen that the vertex is on the Prime Vertical, or 90° from where the great circle crosses the equator. This is 25° to the *Eastward* of A, or in longitude 100° W, which is the answer to question (3). (Figure 23).

The latitude of Vertex is readily seen to be about 38° N.

Now for the coordinates of points on the great circle track, the following approximate values are obtained by inspection.

λ	L
130° W	34°
135° W	32.5°
140° W	31°
145° W	29°
150° W	27°
155° W	24.5°

ADDITIONAL GREAT CIRCLE SAILING PROBLEM

GIVEN: Point $A\begin{cases} L_1 & 42° \text{ N} \\ \lambda_1 & 100° \text{ W} \end{cases}$ $B\begin{cases} L_2 & 25° \text{ N.} \\ \lambda_2 & 70° \text{ W.} \end{cases}$

DETERMINE: (1) Initial Great Circle Course.
(2) Great Circle Distance.
(3) Longitude of Vertex.
(4) Latitude of Vertex.
(5) Latitudes and longitudes of points on great circle track, using longitudes which are multiples of 5°, from the vertex.

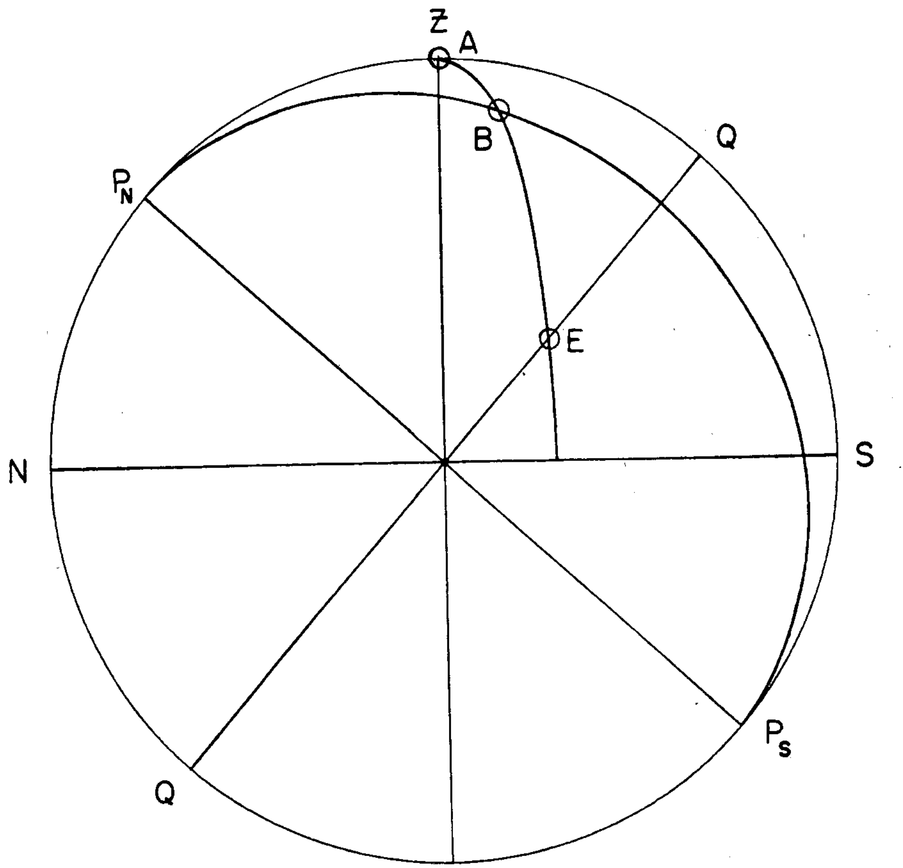

FIG. 24

ANSWERS: (1) 115
(2) 1800 miles
(3) 135° W.
(4) 48° N.
(5) L 40° N λ 95° W
L 38° N λ 90° W
L 36° N λ 85° W
L 33° N λ 80° W
L 30° N λ 75° W
(Figures 24 and 25)

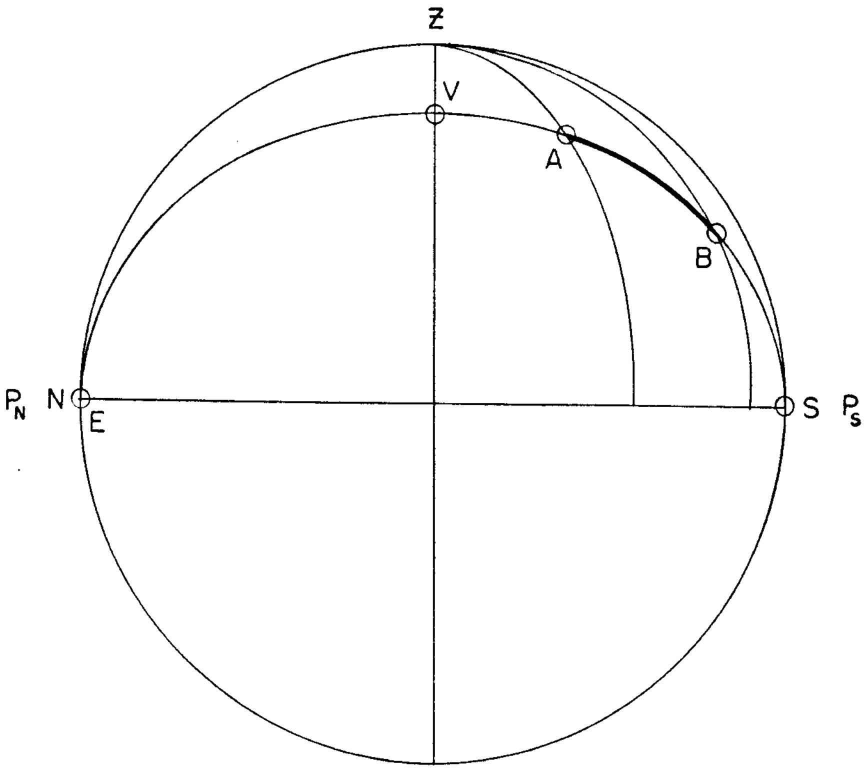

FIG. 25

ANY SPHERICAL TRIANGLE

The Coordinator may be used to obtain complete approximate solutions of spherical triangles with any three parts given. As previously mentioned, the angle at M in navigational problems may be determined by interchanging the values of Latitude and declination, leaving the meridian angle unchanged. With this set-up, the angle at Z is equal to the desired angle at M. This method may be used for the complete solution of any spherical triangle.

A brief description of the best methods to use for the six combinations of given parts, follow:

1. *Given two sides and the included angle.*
   Use the given sides as co-Lat ($\bar{Z}$ Pn) and polar distance (Pn M), and the given angle as meridian angle (angle at Pn).
2. *Given two angles and the included side.*
   Use the given angles as meridian angle and azimuth, and the given side as co-Lat.
3. *Given two sides and the angle opposite one of them.*
   (Possibly two solutions.) Use given side not opposite given angle as polar distance, and the given angle as meridian angle. This locates point M on the equinoctial disc. Rotate the disc until the zenith distance of M is equal to the other given side.
4. *Given two angles and the side opposite one of them.*
   (Possibly two solutions.) Use the given angle not opposite the given side as meridian angle and the given side as polar distance. This locates the point M, on the equinoctial disc. Rotate the equinoctial disc until the vertical circle whose azimuth is equal to the other given angle, meets the point M.

NOTE: It must be remembered that in both cases where a double solution is possible there *may* be only one solution, if the triangle is a right triangle, and no solution at all if the given data are faulty.

5. *Given the three sides.*
   Use the given sides as co-Lat, polar distance, and zenith distance.
6. *Given the three angles.*
   Use the principle of "polar triangles" and proceed as when the three sides are given.

NOTE: If A, B, C, are the angles of a spherical triangle, and a, b, c, the sides opposite, respectively, then in the "*polar*" triangle, $A' = 180 - a$; $a' = 180 - A$, and so forth.

ADDITIONAL PROBLEMS

There follow some additional problems in great circle sailing, for initial great circle course and great circle distance only.

In the following problems "A" is the point of departure and "B" the destination. Determine in each case:

(a) The initial great circle course from A to B.
(b) The great circle distance.
(c) The initial great circle course from B to A. The approximate answers are given.

Problem	Point	Position	Ans.
1.	"A"	L 37½° N, λ 125° W	(a) 250°
	"B"	L 20° N, λ 160° W	(b) 2100 mi. (c) 053°
2.	"A"	L 11½° N, λ 5° E	(a) 040°
	"B"	L 50° N, λ 70° E	(b) 3900 mi. (c) 258°
3.	"A"	L 26° N, λ 100° W	(a) 280°
	"B"	L 25° N, λ 150° W	(b) 2700 mi. (c) 077½°
4.	"A"	L 58½° N, λ 10° W	(a) 110°
	"B"	L 20° N, λ 45° E	(b) 3300 mi. (c) 328°
5	"A"	L 17° S, λ 165° W	(a) 245°
	"B"	L 30° S, λ 130° E	(b) 3600 mi. (c) 093°
6.	"A"	L 53° S, λ 155° W	(a) 080°
	"B"	L 10° S, λ 85° W	(b) 4200 mi. (c) 217°
7.	"A"	L 47½° S, λ 92° E	(a) 310°
	"B"	L 25° S, λ 67° E	(b) 1800 mi. (c) 145°

8. "A"	L 19° S λ 22° W	Ans.
		(a) 120°
"B"	L 35° S λ 28° E	(b) 2820 mi. (c) 276°
9. "A"	L 26½° N λ 42° W	Ans.
		(a) 128°
"B"	L 15° S λ 3° E	(b) 3600 mi. (c) 313°
10. "A"	L 35½° S λ 153° W	Ans.
		(a) 070°
"B"	L 10° N λ 83° W	(b) 4800 mi. (c) 231°
11. "A"	L 30° N λ 20° W	Ans.
		(a) 220°
"B"	L 30° S λ 65° W	(b) 4425 mi. (c) 040°

NOTE: In the last example alone the initial courses are reciprocal.

PROBLEMS

The following problems, with answers, are given for exercise.

1. Given:	A = 97° b = 93½° C = 100°	Ans: a = 96½° B = 95° c = 99½°
2. Given:	a = 99½° B = 114½° C = 82½°	Ans: A = 95½° b = 115½° c = 79°
3. Given:	A = 50° b = 59½° c = 109½°	Ans: a = 69½° B = 45° C = 129½°
4. Given:	A = 55° c = 75° C = 90°	Ans: a = 52½° b = 65° B = 70°
5. Given:	a = 75° b = 38° c = 90°	Ans: A = 65° B = 35° C = 110°

NOTE: Set Latitude at 0°. The side ZPn = 90° = c

6. Given:	a = 151°	Ans: B = 121°	or, B = 59°
	A = 144½°	c = 55½°	c = 24°
	b = 134°	C = 97½°	C = 29°
7. Given:	A = 66°	Ans: a = 74°	
	B = 70°	b = 80°	
	C = 120°	c = 115°	
8. Given:	A = 60°	Ans: a = 70°	or, a = 148°
	b = 50°	c = 93½°	c = 110°
	B = 45°	C = 113°	C = 150
9. Given:	a = 58½°	Ans: A = 44°	
	b = 110°	B = 130°	
	C = 50°	c = 70°	
10. Given:	a = 60°	Ans: A =52½°	or, A = 127°
	b = 50°	c = 87°	c = 15°
	B = 45°	C = 113°	C = 22°
11. Given:	a = 65°	Ans: A = 80°	or, A = 100°
	b = 60°	c = 53°	c = 20°
	B = 70°	C = 60°	C = 21°

Any of the foregoing examples and exercises can be solved on the *stereographic* side of the Coordinator as well. It will be noted that the *orthographic* side gives a better scale near the *center,* while the stereographic is much superior near the *meridian.* The two projections are provided so that any problem may be solved with a good degree of accuracy by employing the appropriate projection.

ANOTHER USE

A "curve" of true azimuth of the sun may be made on the Coordinator by drawing the appropriate declination line for the day in question. Set your watch to local apparent time. Set the Coordinator for the appropriate latitude. Now, the sun's true azimuth can be obtained quite accurately for any given time. Apply the variation of the locality and you have the magnetic azimuth.

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